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Using a Micromechanical Method to Find the Yield Surface of Plasticised Granular Materials

R. Gopalakrishnan, Mr. V. Prakash, Dr. R. Suresh Kumar, Mr. S. R. Ramesh

PROFESSOR [3], ASSOCIATE PROFESSOR [1] ASSISTANT PROFESSOR [2,4]

Department of Mechanical Engineering,

gopalakrishnan.r@actechnology.in, vprakash@actechnology.in, sureshkumar.r@actechnology.in, ramesh.sr@actechnology.in

Arjun College of Technology, Thamaraikulam, Coimbatore-Pollachi Highway, Coimbatore, Tamilnadu-642 120

Abstract: The suggested equation incorporates the fabric and its development to unify the yield surface of granular materials. Microlevel analysis incorporates granular material strength directly using a Fourier series designed for fabric modelling. A noncoaxiality between the deposition angle and primary compressive stress is known as inherent anisotropy. Anisotropy caused by stress is characterised by the angle α and the main direction of the contact normals. This formula describes the discrepancy between samples with the same void ratio (density) but different bedding orientations. When compared to experimental data, the formulation's validity is confirmed.

Introduction

Numerous experimental findings (e.g., [1-3]) demonstrate that the microstructural arrangement (or fabric) influences the morphology of the failure surface for soils. It has long been recognised that the microstructural arrangement of the component particles influences the failure situation. Incorporating fabric's impact and development into failure criteria has been suggested in many formulations. The so-called joint isotropic stress invariants and suitable anisotropic tensorial entities were first up by Baker and Desai [4]. In order to explain fabric anisotropy, Pastor [5] used this technique to suggest a constitutive model. According to Pietruszczak and Mroz [6], the microstructural organisation within the typical volume of material is connected to intrinsic anisotropy. They made use of a tensor of second order, the eigenvectors of which define the orientation of the material symmetry axis. In their proposal, Pietruszczak and Mroz [6] used a microstructure tensor and the stress state to establish the failure criterion. Following the approach put forward by Pietruszczak and Mroz [6], Lade [3] connected the loading directions to the main axes of the particles' cross-anisotropic microstructure arrangements.
As an example, the fabric of anisotropy was described by Oda [1], Oda et al. [2], and Oda [7] utilising the distribution of the unit contact normals, in order to link the microscopic character of the granular materials with overall macroscopic anisotropy. To further understand the relationship between these parameters and the total stress as well as other mechanical properties of granular materials, Mehrabadi et al. [8] established an alternative microstructural arrangement. By building on an isotropic failure criteria and adding two variables to account for fabric anisotropy, Gao et al. [9] and Gao and Zhao [10] suggested a generalised anisotropic failure criterion. Two concepts have been introduced by Oda and Nakayama [11]: the fabric anisotropy and the joint invariants of the deviatoric stress tensor and the deviatoric fabric tensor, which describe the relationship between the direction of stress and fabric anisotropy. They established a connection between the anisotropic variable and the frictional coefficient $\eta k. A$. Friction angle differs in isotropic and anisotropic situations, as shown by Fu and Dafalias [12]. While the friction angle is constant regardless of direction in an isotropic situation, it

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depends on the bedding angle relative to the shear plane in an anisotropic one (as per the Mohr-Coulomb failure criteria). Utilising the discrete element technique (DEM), Fu and Dafalias [13] examined how fabric impacted the shear strength of granular materials. Based on the fact that the orientation of the bedding plane and the shear plane are not coaxial, they put forward an anisotropic shear failure criteria. Thinking about the bedding plane's direction in relation to the main stress axes allowed us to account for the inherent fabric anisotropy.

There have been several suggested criteria for anisotropic granular soils, however the issue of specifying the condition at failure remains significant. In this work, we try to account for the yield surface impact of intrinsic and induced anisotropy. The bedding angle β and the amount of anisotropy β are the explicit functions that describe the inherent and induced anisotropies, as seen in the distribution of contact normals. The Mohr-Coulomb yield surface, which incorporates the fabric and its development, is adjusted to account for the kinematic yield surface [14–16] by combining the two factors, inherent and induced anisotropy. Experimental findings from Oda et al. [17] are contrasted with the suggested yield surface equation for granular soils. Soils with varying bedding angles may have their shearing behaviour captured by the equation.

Definition of Inherent Anisotropy

Inherent anisotropy is attributed to the deposition and orientation of the long axes of particles [1, 2, 7]. Oda et al. [17] and Yoshimine et al. [18] showed that the drained and undrained response of sand and approaching the critical state failure are actually affected by the direction of the principal stress relative to the orientation of the soil sample. Pietruszczak and Mroz [6] included the effect of fabric by the following equation:

$$F = \tau - \eta g(\theta)\, p_o,$$

whereτ=J1/2 2 is the second invariant of the stress tensor, po = trσ/3 is first invariant of the stress tensor, (θ) is Lode's angle, and η is a constant for isotropic materials and defined by the following equation for anisotropic materials:

$$\eta = \eta_o\left(1 + \Omega_{ij} l_i l_j\right),$$

The constant material parameter is denoted by ηo, the bias in the spatial distribution of the material microstructure is described by ib, and the loading directions are li and lb. A failure criteria for anisotropic materials was given by Lade [3] utilising these formulas. The impact of intrinsic anisotropy on microlevel analysis was taken into consideration by Wan and Guo [19] via the ratio of the projected major-to-minor primary values of the fabric tensor along the main stress directions. The fabric tensor, first suggested by Oda and Nakayama [11], was used by Li and Dafalias [20, 21] to integrate this effect. Both approaches started with the same building block: formulating the fabric tensor using its primary values. Micromechanical investigations [2, 11] have shown, however, that granular masses may experience relatively minor shifts in particle preference during shearing. The fabric anisotropy determines where the critical state line is since its value may remain even after the critical state begins. This work models the impact of intrinsic anisotropy using cos $2(\beta i - \beta\circ)$. The particle axis' variation with respect to the main primary stress is denoted by βi, and the angle of deposition with regard to the major principal stress is denoted by $\beta\circ$. Hence,

$$\left(\frac{\sigma_1}{\sigma_2}\right)_f \propto \cos 2\left(\beta_i - \beta_\circ\right).$$

Definition of Stress-Induced Anisotropy

The contact normals incline to congregate in the direction of the primary compressive stress as shear pressures increase. Compressive forces create contacts, whereas tensile forces break them apart. The fundamental reasons for the induced anisotropy in granular materials are these normalisation disruptions and normalisation generating processes [2]. Fabric evolution, also known as induced anisotropy, can only be addressed by defining a function that takes into account changes to the contact normals. The equation that was used by Wan and Guo [19] is:

$$\dot{F}_{ij} = x\dot{\eta}_{ij},$$

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where$F\ ij$ shows the evolution of fabric anisotropy, x is a constant, and $\eta\ ij$ is the ratio of the shear stress to the confining pressure, or $\eta = (q/p)$. Dafalias and Manzari [22] related the evolution of fabric to the volumetric strain in the dilatancy equation. The evolution of fabric comes to play only after dilation. Based on DEM simulation presented by Fu and Dafalias [12], Li and Dafalias [23] developed an earlier model (yield surface) to account for fabric and its evolution in a new manner by considering the evolution of fabric tensor towards its critical value.

By using Fourier series, Rothenburg and Bathurst [24] showed that the contact normals distribution, (n), can be presented as follows:

$$E(n) = \left(\frac{1}{2\pi}\right)\left(1 + \alpha\cos 2\left(\theta - \theta_f\right)\right),$$

whereα is the magnitude of anisotropy and θf is the major principal direction of the fabric tensor. The variations of the parameters α and θf represent the evolution of anisotropy in the granular mass. Experimental data shows that the shear strength of the granular material is a function of the magnitude of α and θf [1, 17, 25]. The following equation is used to consider the effect of the induced anisotropy:

$$\left(\frac{\sigma_1}{\sigma_2}\right)_f \propto \left(1 + \left(\frac{1}{2}\right)\alpha\cos 2\left(\theta_\sigma - \theta_f\right)\right).$$

As previously mentioned, the shear strength in the granular medium is a function of inherent and induced anisotropy. The equation can predict the difference between samples due to the fabric which is a combination of the inherent and induced anisotropy as follows [26]:

$$\left(\frac{\sigma_1}{\sigma_2}\right)_f \propto \left[\left(1 + \left(\frac{1}{2}\right)\alpha\cos 2\left(\theta - \theta_f\right)\right)\cos 2\left(\beta_i - \beta_\circ\right)\right].$$

Another parameter that must be added to the above relation is the rolling strength of the granular material. Oda et al. [25] and Bardet [27] showed the importance of the rolling strength of the particles, especially in a 2D case. This effect is incorporated in the following form [26]:

$$\left(\frac{\sigma_1}{\sigma_2}\right)_f \propto \left[\left(1 + \left(\frac{1}{2}\right)\alpha\cos 2\left(\theta - \theta_f\right)\right) \times \cos 2\left(\beta_i - \beta_\circ\right) m \exp\left(\cos 2\left(\beta_i - \beta_\circ\right)\right)\right],$$

where$m$ is a constant that depends on the interparticle friction angle $\phi\mu$ and the shape of the particles. When the samples with equal densities are subjected to the shear loads, the difference in the shear strength due to the fabric can be attributed to (8).

Verification of (8) with the Experimental Data

By comparing the predictions with the actual tests given by Konishi et al. [25], we can demonstrate that (8) accurately represents the fabric influence on the shear strength. They tested biaxial deformation of two-dimensional assemblies of oval-sectioned rod-shaped photoelastic particles experimentally.A constant force of 0.45 kgf was used to confine the samples laterally, and incremental displacement was used to compress them vertically. The two particle forms that were used were $a1/f2 = 1.1$ and $a1/f2 = 1.4$, where $f1$ and $f2$ are the main and minor axes of the cross section, respectively. In order to examine the impact of friction, two sets of tests were carried out on these two particle shapes. The first set included unlubricated particles with an average friction angle of 52∘, while the second set included lubricated particles with an average friction angle of 26∘. The significance of the anisotropy degree α and the fabric's principal direction θf are determined using the following equations:

$$A = \int_0^{2\pi} E(\theta)\sin 2\theta\, d\theta,$$

$$B = \int_0^{2\pi} E(\theta)\cos 2\theta\, d\theta,$$

$$\theta_f = \left(\frac{1}{2}\right)\arctan\left(\frac{A}{B}\right).$$

To show the ability of (8), the proportion of fabric with the shear strength variations is shown in Figure 1. The differences in the shear strength ratio at failure for different bedding angles are attributed to the differences in the developed anisotropic parameters. In other words, the combination of anisotropic parameters (for inherent and induced anisotropy) is proportional to the shear strength. The variation of righthand side of (8) is

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proportional to the variation of shear strength ratio for different bedding angles. The right-hand side of (8) is shown by fabric anisotropy in Figure 1.The effect of bedding angle on stress ratio at failure for the different interparticle friction angle $\phi\mu$ is also shown in Figure 1.

Incorporation of the Fabric and Its Evolution in the Yield Surface

Muir Wood et al. [14] proposed the kinematic version of the Mohr-Coulomb yield surface as follows:

$$f = q - \eta_y^f p_o,$$

where$q$ is the deviatoric stress and ηfy is the size of the yield surface. Muir Wood et al. [14] and Muir Wood [16] assumed that the soil is a distortional hardening material; hence, the current yield surface ηfy is a function of the plastic distortional strain , and, hence,

$$\eta_y^f = \frac{\varepsilon_q^p}{c + \varepsilon_q^p}\eta^p,$$

whereηp is a limit value of stress ratio which is equal to M at the critical state, $\eta p = M = q/p$; c is a soil constant.

Wood et al. [14] and Gajo and Muir Wood [15] developed the above equation to include the effect of state parameter$\psi = e - e$cr, in which e is the void ratio and $e$cr is the magnitude of the void ratio on the critical-state line, as follows:

$$\eta_y^f = \frac{\varepsilon_q^p}{c + \varepsilon_q^p}(M - k\psi),$$

where$k$ is a constant.

Li and Dafalias [20] modified the effect of state parameter ψ to account for a wide range of stress and void ratio as follows:

$$\eta^p = M \exp(-n_b\psi),$$

In the previous section, the shear strength was shown to be a function of inherent and induced anisotropy (see (8)). Thus, the effect of inherent and induced fabric anisotropy for triaxial case can be expressed as follows:

$$\eta_y^f = \frac{\varepsilon_q^p}{c + \varepsilon_q^p}\left(1 + \left(\frac{1}{2}\right)\alpha\cos 2\left(\theta_f - \theta_\sigma\right)\right) \times \cos 2\left(\beta_i - \beta_\circ\right) M \exp\left(-n_b\psi\right).$$

The magnitudes of α and θf approach a constant value in large shear strain [26, 28, 29]. The parameter cos $2(\beta i - \beta\circ)$ is easily obtained by back calculation but as a rough estimation, its value is close to the magnitude of the bedding angle cosδ (for bedding angle δ between 15∘ and 45∘). Equation (10) can be shown in the following form for multiaxial direction (or in the general form):

$$f = \tau - \eta_y^f g(\theta) p_o.$$

It is similar to the equation proposed by Pietruszczak and Mroz [6] and Lade [3] but in this formulation, another function is used for fabric and its evolution.

Fabric Evolution

The fabric's condition and its development are shown by the parameters α and θb. To a large extent, the dilatancy equation is controlled by these factors. In the condition of noncoaxiality between stress and fabric, Shaverdi et al. [29] presented an equation that may estimate the size of β and θZ. It is from the microlevel analysis that this equation is derived. In order to compute the **ν** parameter, it is necessary to find the size of the shear to normal stress ratio on the spatially mobilised plane (SMP). For instance, in the triaxial situation, one may get τ/k using the equation [30]:

The parameters α and θf may be obtained from the following equations in the presence of noncoaxiality [29]:

$$\alpha = \frac{(\tau/p)\cos\phi_{\mu\text{mob}} - \sin\phi_{\mu\text{mob}}}{\sin\left(2\theta_f + \phi_{\mu\text{mob}}\right) - \left((\tau/p)\cos\left(2\theta_f + \phi_{\mu\text{mob}}\right)\right)},$$

$$\dot{\theta}_f = \dot{\theta}_\sigma + \left(\frac{1}{2}\right)\cdot d\eta\cdot\left(\theta_\sigma - \theta_f\right),$$

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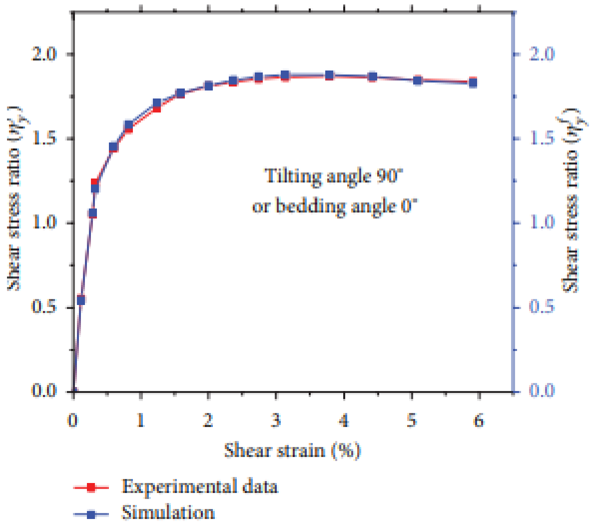
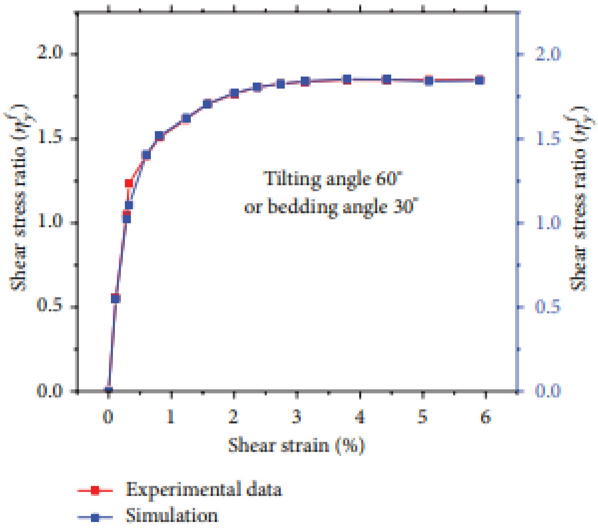
where the dot over θ shows the variation.The most important parameter in the above equation is the interparticle mobilized



(a)



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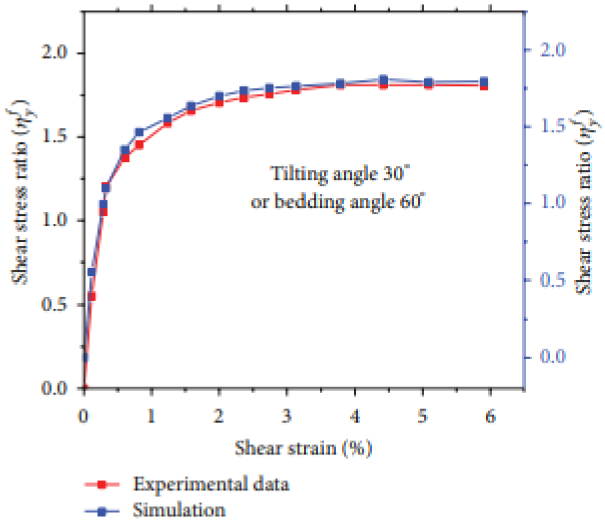

(c)

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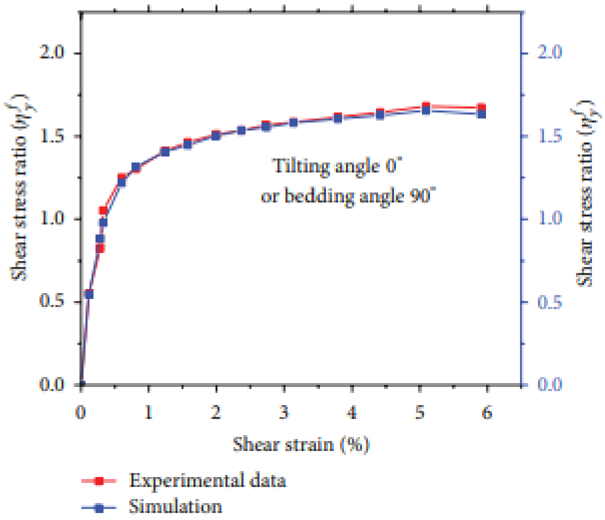

(d)

Figure 2: Comparison between experimental data and simulation by using (16) for the confining pressure 0.5 kg/cm2

Conclusion

The impact of both intrinsic and induced anisotropy was included in a suggested equation. The combination of inherent and induced anisotropy effects yielded this association. This calculation also takes rolling resistance into account. Applying (8) effectively captured the sample differences caused by inherent and induced anisotropy. When applied to granular materials with intrinsic anisotropy, this equation provides the best possible prediction of the ratio of shear strength at failure, as shown by experimental evidence. One term cos $2(\beta i - \beta\circ)$ accounted for the impact of intrinsic anisotropy. An additional simple component that may be readily computed and obtained for α and θf, which is the induced anisotropy, is $(1 + (1/2)\cos 2(\theta f - \theta\sigma))$. To account for the impact of fabric and its development, the extended MohrCoulomb was created. The experimental testing proved that this formulation was valid.

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